



CMWMC 2022, Guts Round, Set 1/8

1. Assume the speed of sound is 343 m/s. Anastasia and Bananastasia are standing in a field in front of you. When they both yell at the same time, you hear Anastasia's yell 5 seconds before Bananastasia's yell. If Bananastasia yells first, and then Anastasia yells when she hears Bananastasia yell, you hear Anastasia's yell 5 seconds after Bananastasia's yell. What is the distance between Anastasia and Bananastasia in meters?
2. Michelle picks a five digit number with distinct digits. She then reverses the digits of her number and adds that to her original number. What is the largest possible sum she can get?
3. Twain is trying to crack a 4-digit number combination lock. They know that the second digit must be even, the third must be odd, and the fourth must be different from the previous three. If it takes Twain 10 seconds to enter a combination, how many hours would it take them to try every possible combination that satisfies these rules?

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4. $\triangle ABC$ is an isosceles triangle with $AB = BC$. Additionally, there is D on BC with $AC = DA = BD = 1$. Find the perimeter of $\triangle ABC$.
5. Let r be the positive solution to the equation $100r^2 + 2r - 1 = 0$. For an appropriate A , the infinite series $Ar + Ar^2 + Ar^3 + Ar^4 + \dots$ has sum 1. Find A .
6. Let $N(k)$ denote the number of real solutions to the equation $x^4 - x^2 = k$. As k ranges from $-\infty$ to ∞ , the value of $N(k)$ changes only a finite number of times. Write the sequence of values of $N(k)$ as an ordered tuple (i.e. if $N(k)$ went from 1 to 3 to 2, you would write $(1, 3, 2)$).

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7. On unit square $ABCD$, a point P is selected on segment CD such that $DP = \frac{1}{4}$. The segment BP is drawn and its intersection with diagonal AC is marked as E . What is the area of triangle AEP ?
8. Five distinct points are arranged on a plane, creating ten pairs of distinct points. Seven pairs of points are distance 1 apart, two pairs of points are distance $\sqrt{3}$ apart, and one pair of points is distance 2 apart. Draw a line segment from one of these points to the midpoint of a pair of these points. What is the longest this line segment can be?
9. The inhabitants of Mars use a base 8 system. Mandrew Mellon is competing in the annual Martian College Interesting Competition of Math (MCICM). The first question asks to compute the product of the base 8 numerals 1245415_8 , 7563265_8 , and 6321473_8 . Mandrew correctly computed the product in his scratch work, but when he looked back he realized he smudged the middle digit. He knows that the product is $1014133027\blacksquare 2766204113_8$. What is the missing digit?



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10. Eve has nine letter tiles: three C's, three M's, and three W's. If she arranges them in a random order, what is the probability that the string "CMWMC" appears somewhere in the arrangement?
11. Bethany's Batteries sells two kinds of batteries: C batteries for \$4 per package, and D batteries for \$7 per package. After a busy day, Bethany looks at her ledger and sees that every customer that day spent exactly \$2021, and no two of them purchased the same quantities of both types of battery. Bethany also notes that if any other customer had come, at least one of these two conditions would've had to fail. How many packages of batteries did Bethany sell?
12. A deck of cards consists of 30 cards labeled with the integers 1 to 30, inclusive. The cards numbered 1 through 15 are purple, and the cards numbered 16 through 30 are green. Lilith has an expansion pack to the deck that contains six indistinguishable copies of a green card labeled with the number 32. Lilith wants to pick from the expanded deck a hand of two cards such that at least one card is green. Find the number of distinguishable hands Lilith can make with this deck.

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13. An equiangular 12-gon has side lengths that alternate between 2 and $\sqrt{3}$. Find the area of the circumscribed circle of this 12-gon.
14. For positive integers n , let $\sigma(n)$ denote the number of positive integer factors of n . Then $\sigma(17280) = \sigma(2^7 \cdot 3^3 \cdot 5) = 64$. Let S be the set of factors k of 17280 such that $\sigma(k) = 32$. If p is the product of the elements of S , find $\sigma(p)$.
15. How many odd 3-digit numbers have exactly four 1's in their binary (base 2) representation? For example, $225_{10} = 11100001_2$ would be valid.

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16. Let x and y be non-negative integers. We say point (x, y) is *square* if $x^2 + y$ is a perfect square. Find the sum of the coordinates of all distinct *square* points which also satisfy $x^2 + y \leq 64$.
17. Two integers a and b are randomly chosen from the set $\{1, 2, 13, 17, 19, 87, 115, 121\}$, with $a > b$. What is the expected value of the number of factors of ab ?
18. Marnie the Magical Cello is jumping on nonnegative integers on number line. She starts at 0 and jumps following two specific rules. For each jump she can either jump forward by 1 or jump to the next multiple of 4 (the next multiple must be strictly greater than the number she is currently on). How many ways are there for her to jump to 2022? (Two ways are considered distinct only if the sequence of numbers she lands on is different.)



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19. The polynomial $x^4 + ax^3 + bx^2 - 32x$, where a and b are real numbers, has roots that form a square in the complex plane. Compute the area of this square.
 20. Tetrahedron $ABCD$ has equilateral triangle base ABC and apex D such that the altitude from D to ABC intersects the midpoint of $\overline{BC}$. Let M be the midpoint of $\overline{AC}$. If the measure of $\angle DBA$ is 67° , find the measure of $\angle MDC$ in degrees.
 21. Last year's high school graduates started high school in year $n - 4 = 2017$, a prime year. They graduated high school and started college in year $n = 2021$, a product of two consecutive primes. They will graduate college in year $n + 4 = 2025$, a square number. Find the sum of all $n < 2021$ for which these three properties hold. That is, find the sum of those $n < 2021$ such that $n - 4$ is prime, n is a product of two consecutive primes, and $n + 4$ is a square.
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22. For monic quadratic polynomials $P = x^2 + ax + b$ and $Q = x^2 + cx + d$, where $1 \leq a, b, c, d \leq 10$ are integers, we say that P and Q are *friends* if there exists an integer $1 \leq n \leq 10$ such that $P(n) = Q(n)$. Find the total number of ordered pairs (P, Q) of such quadratic polynomials that are friends.
23. A three-dimensional solid has six vertices and eight faces. Two of these faces are parallel equilateral triangles with side length 1, $\triangle A_1A_2A_3$ and $\triangle B_1B_2B_3$. The other six faces are isosceles right triangles— $\triangle A_1B_2A_3$, $\triangle A_2B_3A_1$, $\triangle A_3B_1A_2$, $\triangle B_1A_2B_3$, $\triangle B_2A_3B_1$, $\triangle B_3A_1B_2$ —each with a right angle at the second vertex listed (so for instance $\triangle A_1B_2A_3$ has a right angle at B_2). Find the volume of this solid.
24. The digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are each colored red, blue, or green. Find the number of colorings such that any integer $n \geq 2$ has that
 - (a) If n is prime, then at least one digit of n is not blue.
 - (b) If n is composite, then at least one digit of n is not green.